Laser spectroscopy with rubidium

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Abstract This report deals with FM spectroscopy of a rubidium gas cell. A Doppler-broadened and a Doppler-free absorption spectrum is recorded and analyzed. The frequency axis of the spectrum is being calibrated using literature values and by identifying Lamb-dips. Also, the Doppler broadening is determined and discussed. Furthermore, the laser threshold current and absorption coefficient of the *Rb* gas is estimated.

1 Spectroscopy

Spectroscopy is a widely used technique for stabilizing lasers in ultra-cold temperature physics or atomic clocks and for astrophysics as well. By using saturated absorption spectroscopy the transitions from the $5^2S_{1/2}$-state $|1\rangle$ to the $5^2P_{3/2}$-state $|2\rangle$ in Rubidium (^{85}Rb and ^{87}Rb) can be analyzed. Therefore a laser beam with an oscillating frequency passes through a rubidium gas cell exciting electrons from $|1\rangle$ to $|2\rangle$ by getting absorbed. Then the absorption spectrum is measured by a photodiode. This signal leads to the transition frequency of $|1\rangle$ and $|2\rangle$.

Parameter	Fitted value
Slope U	$(65{,}9 \pm 0{,}8)\,\mathrm{mV}$
Threshold current I_T	$(35{,}2 \pm 0{,}6)\,\mathrm{mA}$
χ^2/d_f	5,89

Table 1 Parameters of the linear fit

2 Estimation of the laser threshold

Firstly the output power of the laser was measured as shown in *figure 1*. The measurement was done with the power meter *FieldMaxII* which has an accuracy [1] of ±1%. In the fit only measurements with a laser current between 50 mA and 100 mA were involved, because of saturation effects visible at 130 mA.
For the linear part the fit function

$$P(I) = U * (I - I_T)$$

is used and depends on two parameters: the slope U and the intersection I_T with the I-axis. The threshold current is described by this intersection. A constant offset of $3 \cdot 10^{-4}$ mW was measured, but is negligible because of its magnitude.

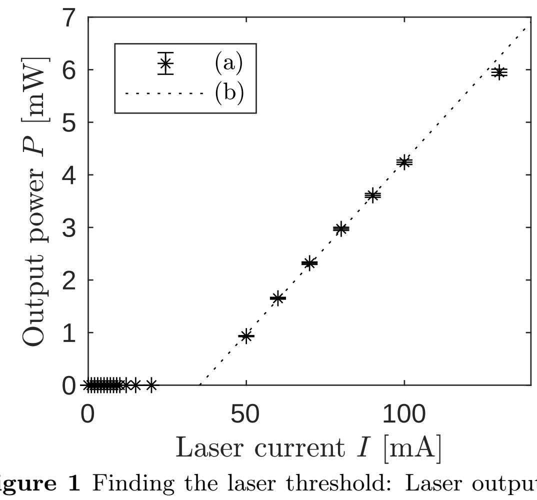

Figure 1 Finding the laser threshold: Laser output power as a function of laser current – (a) measured values, (b) fitted function.

As shown in *table 1* the fit yields a value of the threshold current of:

$$I_T = (35{,}2 \pm 0{,}6)\,\mathrm{mA}$$

3 Doppler-broadened absorption spectrum

In the following section, the Doppler-broadened absorption spectrum of ^{85}Rb and ^{87}Rb is analyzed. Specifically,the D_2-line ($F = 2$ ground state of ^{87}Rb and $F = 3$ ground state of ^{85}Rb) is investigated.

3.1 Setup

Starting with the laser, the beam passes several mirrors (see *figure 2*) and then the rubidium cell. Thereafter it is guided by mirror M3 to the detector. Mirror M3 makes it easier to adjust the beam to hit the detector.

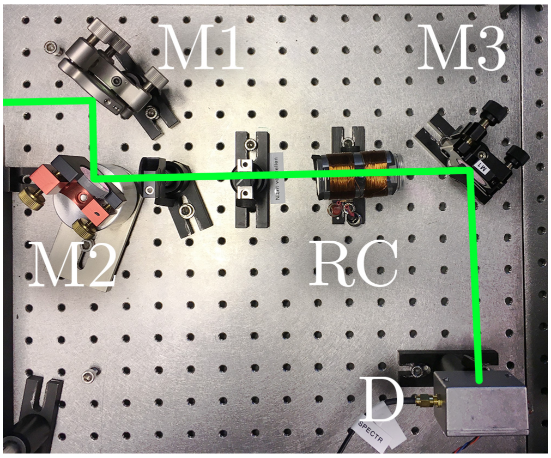

Figure 2 Setup for the Doppler-broadened absorption spectrum. The green line represents the path of the beam. M1, M2, M3 : Mirrors - RC: Rubidium cell - D: Photo detector

Eventually, the signal of the photo detector is displayed on an oscilloscope.

3.2 Theory

Due to the Brownian motion of the gas atoms, the lines of the absorption spectrum are broadened by Doppler-shift. This broadening has a Full Width at Half Maximum of

$$\Delta f_{1/2} = 2\sigma\sqrt{2ln(2)} = 2\frac{f_0}{c}\sqrt{\frac{2k_bT}{M_{Rb}}ln(2)} \tag{1}$$

which for rubidium is approximately 500 MHz. Here σ is the standard deviation for the absorption probability as a function of laser frequency, f_0 is the resonance frequency, c speed of light, k Boltzmann constant, T temperature and M_{Rb} atomic mass of rubidium.

Due to this broadening, the smaller hyperfine structure of the studied D_2-line for rubidium cannot be resolved.

3.3 Measured data & FWHM

The absorption spectrum for the $F = 2 \rightarrow F'$ transition of ^{87}Rb is shown in *figure 3*. It was visible with a laser current of 120,0 mA.

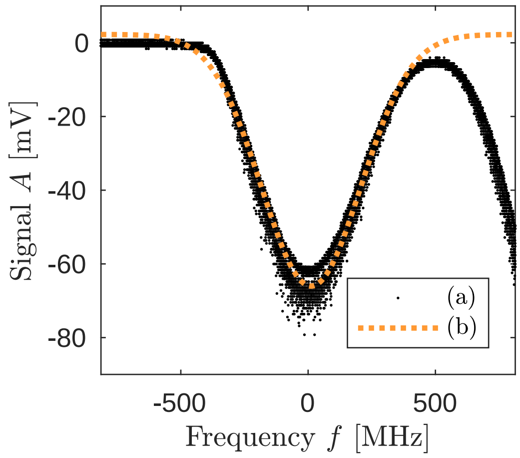

Figure 3 Doppler-broadened absorption spectrum ^{87}Rb, F=2 transition. (a) Measured data - (b) Gaussian fit

To get an approximation of the FWHM a Gauss curve was used as a fit function:

$$G(f) = \frac{G_0}{\sqrt{2\pi}\sigma}\exp\left(-\frac{f-f_0}{2\sigma^2}\right) + \text{const.}$$

The first fit yields a value of the standard deviation of $\sigma = (195{,}0 \pm 0{,}5)$ MHz. This leads to an FWHM of

$$\Delta f_{1/2} = (459 \pm 1)\,\text{MHz}\,.$$

The absorption spectrum for the $F = 3 \rightarrow F'$ transition of ^{85}Rb is shown in *figure 4*. It was visible with a laser current of 121,0 mA.

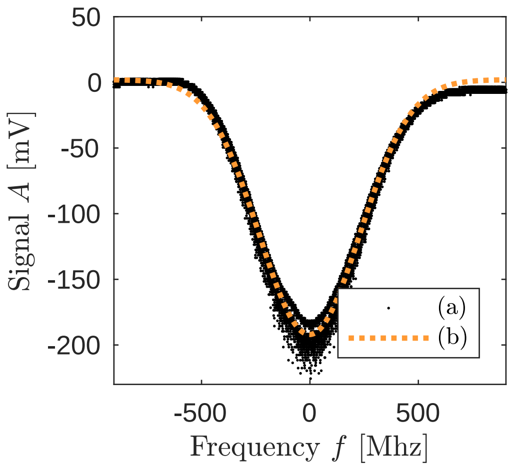

Figure 4 Doppler-broadened absorption spectrum ^{85}Rb, F=3 transition. (a) Measured data - (b) Gaussian fit

The second fit yields a value of the standard deviation of $\sigma = (232{,}1 \pm 0{,}3)\,\mathrm{MHz}$. This leads to an FWHM of

$$\Delta f_{1/2} = (546{,}6 \pm 0{,}5)\,\mathrm{MHz}\,.$$

4 Doppler-free absorption spectrum

A pump and probe technique can be used to differentiate between the hyperfine transitions. Here only the $F = 2$ transitions of ^{87}Rb will be analyzed.

4.1 Setup

As shown in *figure 5* the pump laser beam (green) passes through the *Rb* cell (RC) and a $\lambda/2$ plate (LH) before it is reflected by mirror M3. The pump beam becomes the probe beam (orange) and because of its opposite polarization due to the $\lambda/2$ plate it gets reflected by the beam splitter (BS) to mirror 4 and finally the photo detector (PD).

4.2 Theory

If the laser frequency equals the resonance frequency the pump beam excites most *Rb* atoms with zero velocity. The probe beam which addresses the same (now already excited) atoms experiences fewer losses. A Lamb-dip will be visible.

Exactly between two resonance frequencies a Lamb-dip will be also visible. The pump and probe beam excite atoms of the same velocity class again. But now the beams fulfill different atomic transitions.

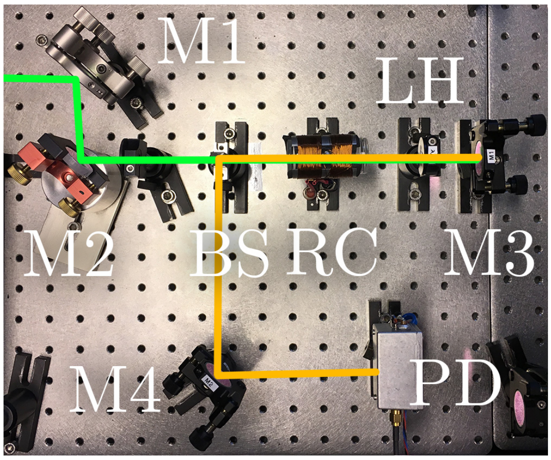

Figure 5 Setup for the Doppler-free absorption spectrum. The green/orange line represents the path of the beam. M1-M4 : Mirrors - RC: Rubidium cell - PD: Photo detector - BS: Beam splitter - LH: Lamda-half plate

Considering the ^{87}Rb $F = 2 \to F'$ transitions and that $\Delta F = 0, \pm 1$ we expect 3 Lamb-dips for the resonance frequencies and 3 for the crossover frequencies. The energy difference between these states is shown in *table 2*.

Momentum F'	Energy difference
3	0
2	266,6 MHz
1	423,5 MHz

Table 2 Difference in energy for the ^{87}Rb $F = 2 \to F'$ transitions compared to the $F' = 3$ transition.

4.3 Measured data & calibration

Regarding the absorption spectrum in *figure 6* just three Lamb-dips are visible. Comparing the image to literature [3] the peaks can be identified. The last one (blue) is coming from the $F' = 3$ transition. The other two peaks describe crossover frequencies.

For the calibration two peaks and their energy difference in terms of frequency is needed. Here the peak of $F' = 3$ and the peak for the crossover frequency between $F' = 3$ and $F' = 2$ is used (the highest peak). Considering the scanning distance between these was 0,1475 ms this leads to a calibration factor of

$$c = \frac{266{,}6\,\mathrm{MHz}}{0{,}295\,\mathrm{ms}} \approx 904\frac{\mathrm{MHz}}{\mathrm{ms}}$$

Thereafter the other lines were plotted taking the literature values into account. Looking closer at the error signal - which describes the negative derivative of the absorption spectrum - the placement of the other lines (CO13, $F' = 2$, CO12) fit to the data. It is interesting that the peaks of the two last lines ($F' = 2$, CO12) cannot be seen in the absorption spectrum but can be recognized in the error signal.

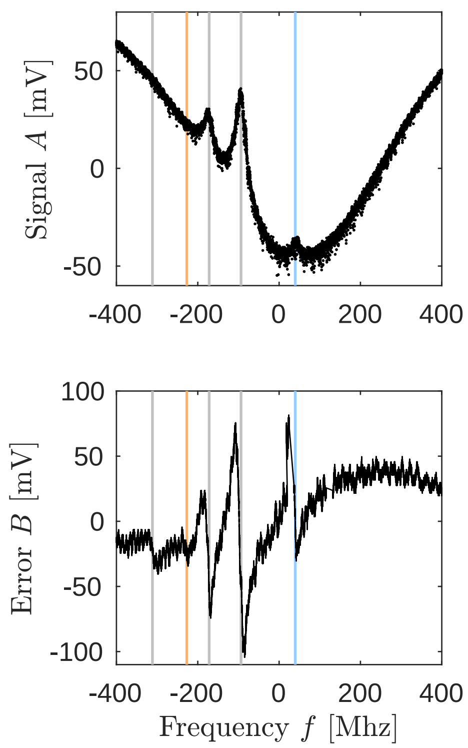

Figure 6 Doppler-free absorption spectrum ^{87}Rb, F=2 transition. - (top) absorption spectrum - (bottom) error signal. - blue: $F' = 3$, orange: $F' = 2$, grey: crossover frequencies CO12, CO13, CO23 (ltr.)

Looking closely at the peak positions and comparing them to the lines the uncertainty regarding c can be estimated to be about 20 MHz/ms.

5 Numerical error signal

To create a numerical error signal the derivative of the absorption spectrum is needed. Here a simple moving filter was used to smooth both spectrum and resulting derivative.

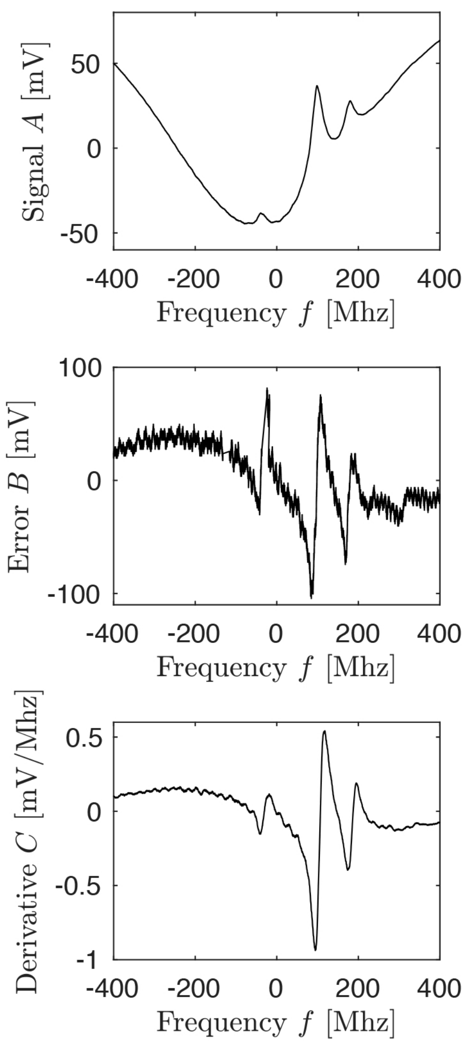

Figure 7 Doppler-free absorption spectrum ^{87}Rb, F=2 transition. - (top) filtered absorption spectrum - (center) error signal - (bottom) numerical error signal

Looking at *figure 7* the overall shape of B and C is similar. The peak at approximately -20 Hz is stronger in the original error signal than in the numerical derivative. Furthermore, the smaller peaks which are slightly visible in the original cannot be found in the numerical error signal. All in all the original error signal provides more information in the detail.

6 Absorption coefficient of *Rb* gas

In order to estimate the absorption coefficient α of the *Rb* gas cell the laser beam power is measured before (P_1) and after (P_2) the cell. Errors will not be included in the following calculations for simplification.
For the measurement, the power meter *FieldMaxII* is used again. Between the beam power P and intensity I the following relationship exists:

$$I = \frac{P}{A} = \frac{P}{\pi d^2} \approx 31{,}8 kg 31{,}8\,\mathrm{kg} \cdot \frac{P}{\mathrm{cm}^2}$$

A describes the area perpendicular to the beam propagation and d the beam diameter which is approximately 1 mm.

	P_1 [μW]	P_2 [μW]	$T = \frac{P_2}{P_1}$	$I_{1/2}$ [$\frac{\mathrm{mW}}{\mathrm{cm}^2}$]
a	630	458	0,73	17,1
b	380	270	0,71	10,2
c	640	445	0,70	17,0

Table 3 Calculating half-way intensity $I_{1/2}$ and transmittance T of the *Rb* gas cell for different transitions. – P_1: Laser beam power before and P_2: after cell. – a/b: transition of ^{85}Rb F=3 – c: transition of ^{87}Rb F=2

The intensity of a beam propagating in direction z and being absorbed by a medium can be described by

$$I(z) = I_1 \cdot e^{-\alpha z}$$

Using $z = d$ with the length d of the *Rb* cell and $T = e^{-\alpha d}$ it follows for the Intensity $I_{1/2} = I(d/2)$

$$I_{1/2} = I_1 \cdot \sqrt{T}\,.$$

Knowing the length of the gas cell $d = 5\,\mathrm{cm}$ it follows for the absorption coefficient α:

$$\alpha = -\frac{\ln T}{d} \approx 7 \frac{1}{\mathrm{m}}$$

	$T = \frac{P_2}{P_1}$	α [$\frac{1}{\mathrm{m}}$]
a	0,73	6,4
b	0,71	6,8
c	0,70	7,3

Table 4 Calculating the absorption coefficient α for different transitions.- T: Transmittance - a/b: ^{85}Rb, F=3 with different light intensity- c: ^{87}Rb F=2

7 Beam walking

The final task was to practice beam walking and beam adjustments. Therefore a fixed target is set up. Two mirrors (see *figure 8*) are used to guide the beam through two pinhole apertures (PA).

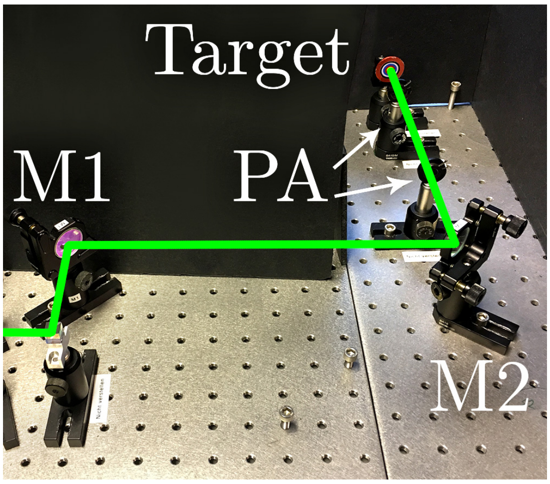

Figure 8 Setup for the beam walking exercise. The green line represents the path of the beam. M1-M2 : Mirrors - PA: Pinhole aperture

Firstly, the mirrors were roughly adjusted. Thereafter M1 was changed to hit the first pinhole perfectly. Consequently, the beam was changed slightly. Now mirror M2 had to be adjusted to hit the second pinhole. This change led to the beam not passing through the first pinhole perfectly anymore. Again M1 had to be adjusted, and so on.

8 Conclusion

8.1 Estimation of laser threshold

The resulting fit yields a value of the threshold of 35,2 mA with an uncertainty of under 2%. The value of the chi-squared test with 5,89 indicates that the measurement uncertainty (1%) might be too low. That makes sense considering that no random error in measuring was minded. Nevertheless, *figure 1* and the result indicate that using a linear fit function is sensible for getting a good idea of the laser threshold current.

8.2 Estimation of FWHM for Doppler-broadened spectrum

For the ^{87}Rb spectrum $\Delta f = (459 \pm 1)\,\mathrm{MHz}$ and for the ^{85}Rb spectrum $\Delta f = (546{,}6 \pm 0{,}5)\,\mathrm{MHz}$ is calculated. Looking at *figure 3 and 4* the Gaussian fit is a good way to describe the measured data for an estimation of the FWHM. This is also indicated by an error of under 0,3% regarding the fit results. Considering the inexactness of the calibration (see *section 4.3*) the uncertainty of Δf should be around 10 Mhz.

The theoretical value for the FWHM taking (1) and $kT = 25{,}7\,\mathrm{meV}$, $M = 85{,}5\,\mathrm{u}$ and $\lambda_0 = 780{,}24\,\mathrm{nm}$ is:

$$\Delta f \approx 514{,}5\,\mathrm{MHz}$$

The theoretical value lies between the experimental ones. Both calculated values were expected to be higher than 514,5 MHz due to neglected disturbances in theory. It is interesting that the FWHM for the ^{87}Rb spectrum is about 10% smaller than the theoretical value.

8.3 Doppler-free spectrum and calibration

To see the Lamb-dips it is important to make sure that probe and pump beam overlay as good as possible. Even so the signal of the Lamb-dips ($F' = 1{,}2{,}3$) is not very strong. Also, the error signal is quiet noisy. Nevertheless, it was possible to identify the visible peaks and thus calibrate the frequency axis.

8.4 Estimation of absorption coefficient

To measure the laser beam power before and after the *Rb* cell, the position of the photo detector was changed. Hitting the detector with the laser in the same way is impossible. To minimize the error, the detector was adjusted in a way that the power value was at a maximum. This source of error might be lessened by moving the *Rb* cell instead of the photo diode. One could also place a vacuum cell with the same size as the rubidium cell in front of the detector.

Another fact to note is that transition c was expected to have a higher transmittance T than a or b. Looking at *figure 3 and 4* the absorption signal for ^{85}Rb is much stronger than the one of ^{85}Rb . This might be due to the fact that the laser frequency modulation range is large and the peak of the neighbor transition was scanned as well. Also, it could be an indication of the large measurement uncertainties.

References

[1] FieldMaxII power meter manual [online]. URL: `https://goo.gl/pKRQpX`.

[2] W. Demtröder. *Experimentalphysik 1*. Experimentalphysik / Wolfgang Demtröder. Springer, 2006.

[3] Aline Dinkelaker, Marek Mandel, and Sana Pyka. Laser spectroscopy with rubidium [online]. May 2017. URL: `https://www.physics.hu-berlin.de/en/qom/lehre/f-praktikum-laserspektroskopie/anleitung.pdf`.

9 Appendix

9.1 Records

Laser spectroscopy with rubidium

Max Dreyer
Hannes Vogel

1. Measuring the output power of the laser

↙ u = 1%

output power [W]	326 320	320 323 ±5	314 323	310 326	311 330	318	328	340
laser current [mA]	0	1	2	3	4	5	6	7

280 273 353	365	380	280 280 260 420	282 280 510	290 280 270 304 830n	933μ	1,653m
8	9	10	12	15	20	50	60

2. Doppler-broadened absorption

2,3 20 m	2,973m	3,61m	4,24m	5,95m
70	80	90	100	130

(I) Save absorption spectrum and plot on computer

(II) Save .... 85F3E01 for ^{87}Rb F=2

laser current: 120,0mA

↕

(III) Save ... 85F3E02 / 85F3E03 for ^{85}Rb F=3

laser current: 121,0mA 8

3. Doppler-free setup

(I) 87 F=2 → 85F3E(04) (spectrum) I = 121,6mA (4)
05 error
07 spectrum
08 error

(II) 85 F=3 06 spectrum I = 120,7mA (3)
07 error

3. Measure laser beam power before and after cell.

before: $P = 0{,}630\ mW$ $\qquad u = 1\times$ $\qquad$ 10: error

9: signal $\qquad$ 85

after: $P = 0{,}458\ mW$

before: $P = 0{,}640\ mW$ $\qquad$ 87

after: $P = 0{,}445\ mW$

before: $P = 0{,}380\ mW$

after: $P = 0{,}270\ mW$ $\qquad$ 85